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Convex large deviation rate functions under mixtures of linear transformations, with an application to ruin theory

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Abstract

Let $X_1, X_2, \ldots$ be a sequence of random vectors taking values in $\mathbf{R}^d$. Let $\mathcal{A}$ be a random $d' \times d$ matrix which is independent of the process $\{X_n\}$. Suppose that $\{X_n\}$ satisfies the large deviations upper or lower bounds with a convex rate function. Starting with this, we derive large deviations statements for the mixture $\{\mathcal{A}X_n\}$. The case where $\mathcal{A}$ is deterministic is studied in more detail in the framework of the Gärtner–Ellis theorem. The results are applied to a ruin problem.

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1. Introduction

Let d and d' be positive integers and $\{X_n\}$ a sequence of random vectors taking values in $\mathbf{R}^d$. Let $\mathcal{A}$ be a random $d' \times d$ matrix which is independent of $\{X_n\}$. By considering X_n as a column vector, we can study the mixture $\{\mathcal{A}X_n\}$. Our objective is to derive large deviations properties for $\{\mathcal{A}X_n\}$ from large deviations properties of $\{X_n\}$.

To describe our interest in more detail, we recall some basic concepts from large deviations theory. A function $I : \mathbf{R}^d \to \mathbf{R} \cup \{+\infty\}$ is a *rate function* if it is non-negative and lower semicontinuous. The sequence $\{X_n\}$ satisfies the *large deviations upper bounds* with the rate function I if

$$\limsup_{n\to\infty} n^{-1} \log \mathbf{P}(X_n \in F) \leq -\inf\{I(x); x \in F\} \tag{1.1}$$

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for every closed set $F \subseteq \mathbf{R}^d$ and *large deviations lower bounds* with the rate function I if

$$\liminf_{n\to\infty} n^{-1} \log \mathbf{P}(X_n \in G) \geq -\inf\{I(x); x \in G\} \tag{1.2}$$

for every open set $G \subseteq \mathbf{R}^d$. The sequence $\{X_n\}$ satisfies the *large deviations principle* with the rate function I if it satisfies both the large deviations upper and lower bounds with the rate function I.

Our interest is in the case where I is convex. Then it can be represented as the Fenchel–Legendre transform of another convex function which is often understood by other means. This is very concrete in the Gärtner–Ellis theorem but holds true in general. See Gärtner [9], Ellis [7], O'Brien and Vervaat [16] and Dinwoodie [6] for the background. There are many interesting processes which satisfy the large deviations principle with a convex rate function. A wide class is given in the above mentioned Gärtner–Ellis theorem. We refer the reader to Dembo and Zeitouni [4] for more information about large deviations theory.

Let A be a deterministic $d' \times d$ matrix. Given that (1.1) holds for every closed set or that (1.2) holds for every open set, it is possible to derive similar statements for the sequence $\{AX_n\}$ by means of the contraction principle. The resulting rate function is convex so that it has a representation as the Fenchel–Legendre transform of a convex function. We will focus on this representation. It can be used to see that the large deviations structure of the Gärtner–Ellis theorem is typically preserved under linear transformations. It turns out that the above descriptions can be given for every $d' \times d$ matrix. This provides a natural starting point for extending the study to be concerned with random matrices. Useful background results in this area are given in Dinwoodie and Zabell [5].

The rest of the paper is organized as follows. We begin in Section 2 with the general case where $\mathcal{A}$ is random. The results will be specified in Section 3 for deterministic matrices. An application to ruin theory is presented in Section 4. Section 5 consists of the proofs. The theory of convex functions is very useful for our study. Necessary background results for this part are recalled in the Appendix.

2. Main results for mixtures

Let $(\Omega, S, \mathbf{P})$ be a probability space and d and d' positive integers. Let $\{X_n\}$ be a sequence of random vectors on the measurable space (Ω, S) taking values in $\mathbf{R}^d$ and let $\mathcal{A}$ be a random $d' \times d$ matrix on (Ω, S). We assume that $\mathcal{A}$ is independent of X_n for every n. Denote by Q the distribution of $\mathcal{A}$ and by $\mathcal{S}$ the support of Q. Then we have

$$\mathbf{P}(\mathcal{A}X_n \in B) = \int_{A\in\mathcal{S}} \mathbf{P}(AX_n \in B)\mathrm{d}Q(A) \tag{2.1}$$

for every $n \in \mathbf{N}$ and every Borel set $B \subseteq \mathbf{R}^{d'}$. We are interested in the large deviations of the sequence $\{\mathcal{A}X_n\}$.

The set-up may be seen as a special case of that of Dinwoodie and Zabell [5]. In fact, let $A \in \mathcal{S}$ be an arbitrary deterministic $d' \times d$ matrix and I_A a rate function on $\mathbf{R}^{d'}$. Suppose that for every open set $G' \subseteq \mathbf{R}^{d'}$,

$$\liminf_{n\to\infty} n^{-1} \log \mathbf{P}(A_n X_n \in G') \geq -\inf_{y\in G'} I_A(y) \tag{2.2}$$

whenever $\{A_n\}$ is a sequence of matrices tending to A. If we have (2.2) for every $A \in \mathcal{S}$ then Theorem 2.1 of Dinwoodie and Zabell [5] provides large deviations lower bounds for the mixture

$\{\mathcal{A}X_n\}$ in terms of the rate functions I_A. Our objective is to identify the rate functions and to apply the result under suitable conditions on the sequence $\{X_n\}$. A continuity property similar to (2.2) implies upper bounds for $\{\mathcal{A}X_n\}$ in the case where $\mathcal{S}$ is compact (Theorem 2.2 of Dinwoodie and Zabell [5]). In the present model, it is possible to derive upper bounds directly. So we do not need any continuity properties for this part. In the case where $\mathcal{S}$ is not compact, our approach for the upper bounds is very different from that of Dinwoodie and Zabell [5].

We introduce some concepts and notations to describe our results. More details can be found in the Appendix. Let $B \subseteq \mathbf{R}^d$ be a convex set. Denote by $\mathrm{int}B$, $\mathrm{ri}B$, $\mathrm{cl}B$ and B^c the interior, the relative interior, the closure and the complement of B, respectively. The Euclidean inner product of $x, y \in \mathbf{R}^d$ is denoted by $\langle x, y\rangle$. Let $f : \mathbf{R}^d \to \mathbf{R} \cup \{\pm\infty\}$ be a convex function. The Fenchel–Legendre transform f^* of f is defined by

$$f^*(x) = \sup\{\langle \lambda, x\rangle - f(\lambda); \lambda \in \mathbf{R}^d\}$$

for $x \in \mathbf{R}^d$. It is a convex and lower semicontinuous function on $\mathbf{R}^d$. Denote by $\mathrm{dom}\, f$ the effective domain and by $\mathrm{cl} f$ the closure of f. Let $g : \mathbf{R}^{d'} \to \mathbf{R} \cup \{\pm\infty\}$ be a convex function and let A be a $d' \times d$ matrix. The function $gA : \mathbf{R}^d \to \mathbf{R} \cup \{\pm\infty\}$ is defined by

$$(gA)(x) = g(Ax). \tag{2.3}$$

Then gA is also convex. Finally, denote by A^{T} the transpose of A.

We will assume in the sequel large deviations properties for $\{X_n\}$ with the rate function Γ^* where Γ is a convex function. Large deviations of the sequence $\{\mathcal{A}X_n\}$ can then be controlled by the function I_1 defined by

$$I_1(y) = \inf_{A \in \mathcal{S}} \left((\mathrm{cl}\Gamma)A^{\mathrm{T}}\right)^* (y) \tag{2.4}$$

for $y \in \mathbf{R}^{d'}$. We also consider its simplification, namely, the function I_2 defined on $\mathbf{R}^{d'}$ by

$$I_2(y) = \inf_{A \in \mathcal{S}} \left(\Gamma A^{\mathrm{T}}\right)^* (y). \tag{2.5}$$

Theorem 2.1. *Let $\Gamma : \mathbf{R}^d \to \mathbf{R} \cup \{\pm\infty\}$ be a convex function. Assume that $\{X_n\}$ satisfies the large deviations upper bounds with the rate function Γ^* and that $\mathcal{S}$ is compact. Then for every closed set $F' \subseteq \mathbf{R}^{d'}$,*

$$\limsup_{n\to\infty} n^{-1} \log \mathbf{P}(\mathcal{A}X_n \in F') \le -\inf_{y\in F'} I_1(y) \tag{2.6}$$

and

$$\limsup_{n\to\infty} n^{-1} \log \mathbf{P}(\mathcal{A}X_n \in F') \le -\inf_{y\in F'} I_2(y). \tag{2.7}$$

We next give sufficient conditions under which the compactness of $\mathcal{S}$ is not needed.

Theorem 2.2. *Let the function Γ be as in Theorem 2.1. Assume that $\{X_n\}$ satisfies the large deviations upper bounds with the rate function Γ^*. Assume further that the restriction of Γ^* to* $\mathrm{dom}\,\Gamma^*$ *is continuous and that*

$$\lim_{n\to\infty} n^{-1} \log \mathbf{P}(X_n \in (\mathrm{cl}(\mathrm{dom}\,\Gamma^*))^c) = -\infty. \tag{2.8}$$

Then (2.6) and (2.7) hold for every closed set $F' \subseteq \mathbf{R}^{d'}$.

We note that as a convex function, Γ^* is always continuous relative to $\mathrm{ri}(\mathrm{dom}\,\Gamma^*)$. See Rockafellar [20, Theorem 10.1].

Consider now large deviations lower bounds.

Theorem 2.3. *Let the function Γ be as in Theorem 2.1. Assume that $\{X_n\}$ satisfies the large deviations lower bounds with the rate function Γ^*. Then for every open set $G' \subseteq \mathbf{R}^{d'}$,*

$$\liminf_{n\to\infty} n^{-1} \log \mathbf{P}(\mathcal{A}X_n \in G') \geq -\inf_{y\in G'} I_1(y). \tag{2.9}$$

Assume in addition that for every $A \in \mathcal{S}$, there exists $\kappa_A \in \mathbf{R}^{d'}$ such that $A^{\mathrm{T}}\kappa_A \in \mathrm{ri}(\mathrm{dom}\,\Gamma)$. Then for every open set $G' \subseteq \mathbf{R}^{d'}$,

$$\liminf_{n\to\infty} n^{-1} \log \mathbf{P}(\mathcal{A}X_n \in G') \geq -\inf_{y\in G'} I_2(y). \tag{2.10}$$

Simplified form (2.10) for the lower bounds holds for every random $d' \times d$ matrix $\mathcal{A}$ if, for example, Γ is lower semicontinuous or $0 \in \mathrm{ri}(\mathrm{dom}\,\Gamma)$.

Remark 2.1. Consider the special case where $\mathcal{A}$ is deterministic, $\mathcal{A} = A$, say. Suppose that Γ^* is a good rate function (that is, the set $\{x \mid \Gamma^*(x) \leq \alpha\}$ is compact for every $\alpha \in \mathbf{R}$). If $\{X_n\}$ satisfies the large deviations principle with the rate function Γ^* then, by the contraction principle, $\{AX_n\}$ satisfies the large deviations principle with the rate function J_A where

$$J_A(y) = \inf\{\Gamma^*(x) \mid Ax = y\} \tag{2.11}$$

for $y \in \mathbf{R}^{d'}$. See Theorem 4.2.1 in Dembo and Zeitouni [4]. By the uniqueness of the large deviations rate function and by Theorems 2.1 and 2.3, we have $J_A = \big((\mathrm{cl}\Gamma)A^{\mathrm{T}}\big)^*$. This representation of J_A may be seen to be more elementary than (2.11) because it is defined as the solution of one unconstrained optimization problem. Theorem 2.3 further shows that we often have $J_A = \big(\Gamma A^{\mathrm{T}}\big)^*$.

Remark 2.2. It may happen that I_1 (or I_2) is not lower semicontinuous so that, strictly speaking, (2.6) and (2.9) are not standard statements in large deviations theory. However, if (2.6) holds then it also holds when I_1 is replaced by its lower semicontinuous hull (the greatest lower semicontinuous function majorized by I_1). Similarly, I_1 can be replaced by its lower semicontinuous hull in (2.9). Hence, given that (2.6) and (2.9) hold, then $\{\mathcal{A}X_n\}$ satisfies a standard large deviations principle. This useful observation is made in Orey [17, Proposition 1.1].

We end the section with examples which illustrate the conditions of Theorem 2.2.

Example 2.1. Let $\{Y_n\}$ be a random walk in $\mathbf{R}^d$ and $X_n = n^{-1}Y_n$ for $n \in \mathbf{N}$. Let Γ be the logarithm of the moment generating function of Y_1. That is,

$$\Gamma(\gamma) = \log \mathbf{E}\{\mathrm{e}^{\langle\gamma, Y_1\rangle}\}$$

for $\gamma \in \mathbf{R}^d$. Then Γ is convex. Assume that $\{X_n\}$ satisfies the large deviations principle with the rate function Γ^*. By Cramér's theorem, this is often the case. See Dembo and Zeitouni [4, Theorem 6.1.3]. Example 2.1 of Bahadur and Zabell [2] shows that there exist random walks such that the restriction of Γ^* to $\mathrm{dom}\,\Gamma^*$ is not continuous. We show that (2.8) always holds.

Fix $n_0 \in \mathbf{N}$. We prove that, actually,

$$\mathbf{P}(X_{n_0} \in (\mathrm{cl}(\mathrm{dom}\,\Gamma^*))^c) = 0. \tag{2.12}$$

This certainly implies (2.8). Sufficient conditions for (2.12) are given in part (c) of Thorem 2.4 in Bahadur and Zabell [2].

Consider an open ball $B(x, r) \subseteq (\mathrm{cl}(\mathrm{dom}\,\Gamma^*))^c$. Put $Y_0 \equiv 0$. Because $B(x, r)$ is convex we have for every $k \in \mathbf{N}$,

$$\begin{aligned}\mathbf{P}(Y_{kn_0}/(kn_0) \in B(x, r)) &\geq \mathbf{P}((Y_{in_0} - Y_{(i-1)n_0})/n_0 \in B(x, r) \text{ for } i = 1, \dots, k)\\ &= \mathbf{P}(Y_{n_0}/n_0 \in B(x, r))^k. \end{aligned} \tag{2.13}$$

Consequently,

$$\limsup_{n\to\infty} n^{-1} \log \mathbf{P}(X_n \in B(x, r)) \geq n_0^{-1} \log \mathbf{P}(X_{n_0} \in B(x, r)). \tag{2.14}$$

By Theorem 6.1.3 of Dembo and Zeitouni [4], the left hand side of (2.14) equals $-\infty$. It follows that $\mathbf{P}(X_{n_0} \in B(x, r)) = 0$. The open set $(\mathrm{cl}(\mathrm{dom}\,\Gamma^*))^c$ is a countable union of open balls like $B(x, r)$ above. This implies (2.12).

Example 2.2. Let Γ be as in Theorem 2.2. Assume that $\{X_n\}$ satisfies the large deviations upper bounds with the rate function Γ^* and that $\mathrm{dom}\,\Gamma^*$ is open. Then (2.8) holds even when $\mathrm{cl}(\mathrm{dom}\,\Gamma^*)$ is replaced by $\mathrm{dom}\,\Gamma^*$. By Rockafellar [20, Theorem 10.1], Γ^* is continuous on $\mathrm{int}(\mathrm{dom}\,\Gamma^*) = \mathrm{dom}\,\Gamma^*$. Hence, we have (2.6) and (2.7) for every random $d' \times d$ matrix $\mathcal{A}$.

Example 2.3. We provide a simple process for which condition (2.8) fails. Let $\xi, \xi_1, \xi_2, \dots$ be independent and identically distributed random variables. Suppose that $\mathbf{P}(\xi = 1) = p$ and $\mathbf{P}(\xi = 0) = 1 - p$ where $p \in (0, 1)$. Let

$$X_n = n^{-1}(2\xi_1 + \xi_2 + \cdots + \xi_n) \tag{2.15}$$

for $n \in \mathbf{N}$. Then $\{X_n\}$ satisfies the large deviations principle with the rate function Γ^* where Γ is the logarithm of the moment generating function of ξ. This follows from the Gärtner–Ellis theorem (Theorem 2.3.6 in Dembo and Zeitouni [4]). An easy calculation shows that $\mathrm{cl}(\mathrm{dom}\,\Gamma^*) = [0, 1]$. On the other hand,

$$\mathbf{P}(X_n > 1) \geq \mathbf{P}(\xi_1 = 1, \dots, \xi_n = 1) = p^n \tag{2.16}$$

so that (2.8) does not hold.

3. A connection with the Gärtner–Ellis theorem

Let the process $\{X_n\}$ be as in Section 2 and let A be a deterministic $d' \times d$ matrix. We will apply the results of Section 2 to the sequence $\{AX_n\}$ in the framework of the Gärtner–Ellis theorem. Define the function $\Lambda : \mathbf{R}^d \to \mathbf{R} \cup \{\pm\infty\}$ by

$$\Lambda(\lambda) = \limsup_{n\to\infty} n^{-1} \log \mathbf{E}\{\mathrm{e}^{n\langle\lambda, X_n\rangle}\}. \tag{3.1}$$

Then Λ is convex. The rate function of interest to us is Λ^*.

Theorem 3.1. *If $0 \in \operatorname{int}(\operatorname{dom} \Lambda)$ then $\{X_n\}$ satisfies the large deviations upper bounds with the rate function Λ^*. If Λ is essentially smooth and (3.1) holds as the limit for every $\lambda \in \operatorname{int}(\operatorname{dom} \Lambda)$ then $\{X_n\}$ satisfies the large deviations lower bounds with the rate function Λ^*.*

The above result is in essence the Gärtner–Ellis theorem. We refer the reader to Dembo and Zeitouni [4] and O'Brien and Vervaat [16]. The only difference is that we do not assume any tightness conditions for the lower bounds. The extension is useful in Example 3.1 below, and probably also in other similar situations.

Define the function $\Lambda_A : \mathbf{R}^{d'} \to \mathbf{R} \cup \{\pm\infty\}$ by

$$\Lambda_A(\kappa) = \limsup_{n\to\infty} n^{-1} \log \mathbf{E}\{e^{n\langle \kappa, AX_n\rangle}\}. \tag{3.2}$$

It is the counterpart of (3.1) for the sequence $\{AX_n\}$. An easy calculation shows that $\Lambda_A = \Lambda A^{\mathrm{T}}$. It is seen that linear transformations often preserve the structure of large deviations of the Gärtner–Ellis theorem. For example, if $\{X_n\}$ satisfies the large deviations principle with the rate function Λ^* and $0 \in \operatorname{ri}(\operatorname{dom} \Lambda)$ then $\{AX_n\}$ satisfies the large deviations principle with the rate function $(\Lambda_A)^*$ for every $d' \times d$ matrix A. This follows from Theorems 2.1 and 2.3 on choosing $\mathcal{A} = A$ and $\Gamma = \Lambda$.

Example 3.1. Let $\xi, \xi_1, \xi_2, \ldots$ be independent and identically distributed random variables and $X_n = n^{-1}(\xi_1 + \cdots + \xi_n)$ for $n \in \mathbf{N}$. We prove by means of Theorems 2.3 and 3.1 the lower bounds of Cramér's theorem, namely, that (1.2) holds for every open set $G \subseteq \mathbf{R}$ with $I = \Lambda^*$. A slight modification gives the extension for the multidimensional random walks. We refer the reader to Bahadur and Zabell [2] and De Acosta et al. [1] for earlier proofs.

In the present case, Λ is the logarithm of the moment generating function of ξ. Hence, the result follows from Theorem 3.1 if Λ is finite everywhere. Assume henceforth that $\operatorname{dom} \Lambda \neq \mathbf{R}$. Consider the two-dimensional process $\{\overline{X}_n\}$ defined by

$$\overline{X}_n = n^{-1} \sum_{i=1}^{n} (\xi_i, \exp(\xi_i^2)). \tag{3.3}$$

Corresponding to (3.1), write

$$\overline{\Lambda}(\lambda) = \limsup_{n\to\infty} n^{-1} \log \mathbf{E}\{e^{n\langle \lambda, \overline{X}_n\rangle}\} \tag{3.4}$$

for $\lambda \in \mathbf{R}^2$. Then

$$\overline{\Lambda}(\lambda_1, \lambda_2) = \log \mathbf{E}\left(e^{\lambda_1 \xi + \lambda_2 \exp(\xi^2)}\right) \tag{3.5}$$

and, trivially, (3.4) holds as the limit for every $\lambda_1, \lambda_2 \in \mathbf{R}$. It is not difficult to see that

$$\operatorname{int}(\operatorname{dom} \overline{\Lambda}) = \{(\lambda_1, \lambda_2)^{\mathrm{T}} \in \mathbf{R}^2; \lambda_2 < 0\} \tag{3.6}$$

and that $\overline{\Lambda}$ is essentially smooth. Thus by Theorem 3.1, $\{\overline{X}_n\}$ satisfies the large deviations lower bounds with the rate function $\overline{\Lambda}^*$. The process $\{X_n\}$ is obtained from $\{\overline{X}_n\}$ by a linear transformation. Since $\overline{\Lambda}$ is lower semicontinuous we conclude by Theorem 2.3 that the desired lower bounds hold.

4. An application to ruin theory

Consider an insurance company with the following properties. There are n policy holders who pay premiums to and receive compensations from the company. Let Y_{nj} be the total net payoff of the company in the year $j \in \{1, \ldots, d\}$. That is, Y_{nj} equals the compensations less the premiums. Write $Y_n = (Y_{n1}, \ldots, Y_{nd})^{\mathrm{T}}$ for $n \in \mathbf{N}$. Let $u > 0$ be a constant and let nu be the initial capital of the company. We assume that the capital and the subsequent profits are invested in risky assets. With the year j, we associate the discount factor ξ_j corresponding to the returns on the investments. Assume that $\xi_1, \ldots, \xi_d$ are positive random variables and that they are independent of the process $\{Y_n\}$. Write

$$\mathcal{A} = \begin{pmatrix} \xi_1 & 0 & 0 & \ldots & 0 \\ \xi_1 & \xi_1\xi_2 & 0 & \ldots & 0 \\ \ldots & & & & \ldots \\ \xi_1 & \xi_1\xi_2 & \xi_1\xi_2\xi_3 & \ldots & \xi_1\cdots\xi_d \end{pmatrix}. \tag{4.1}$$

Then we have

$$\mathcal{A}Y_n = \left(\xi_1 Y_{n1}, \xi_1 Y_{n1} + \xi_1\xi_2 Y_{n2}, \ldots, \sum_{j=1}^{d}\left(\prod_{k=1}^{j}\xi_k\right) Y_{nj}\right)^{\mathrm{T}}. \tag{4.2}$$

Hence, $\mathcal{A}Y_n$ shows the present values of the accumulated net payoffs (we have made the simplification that all the insurance payments take place at the ends of the years). Let $T = T(n, u)$ be the time of ruin of the company. That is, T is the first time when the capital of the company is negative. Then we have

$$\{T \le d\} = \bigcup_{m=1}^{d}\left\{\sum_{j=1}^{m}\left(\prod_{k=1}^{j}\xi_k\right) Y_{nj} > un\right\}. \tag{4.3}$$

See Daykin, et al. [3] for the general background for the model and Nyrhinen [13] for the background for representation (4.3).

Let $X_{nj} = n^{-1}Y_{nj}$ for $n \in \mathbf{N}$ and $j \in \{1, \ldots, d\}$, and write $X_n = (X_{n1}, \ldots, X_{nd})^{\mathrm{T}}$. According to Section 2, we assume that $\{X_n\}$ satisfies the large deviations principle with the rate function Γ^* where Γ is convex. This assumption is natural if, for example, the payoffs generated by the policy holders are independent and identically distributed. Results of Section 2 can now be applied to ruin probabilities. In fact,

$$\{T \le d\} = \{\mathcal{A}X_n \in C_u\}$$

where

$$C_u = \{(y_1, \ldots, y_d)^{\mathrm{T}} \in \mathbf{R}^d;\ y_j > u \text{ for some } j \in \{1, \ldots, d\}\}. \tag{4.4}$$

There are many recent studies on such models where risky investments are allowed. We refer the reader to Paulsen [18,19], Nyrhinen [14,15], Frolova, Kabanov and Pergamenchtchikov [8], Kalashnikov and Norberg [10] and Tang and Tsitsiashvili [21,22]. Our viewpoint is different from those of the above papers since we consider limits when the number of policy holders and the initial capital increase. This can be seen as a big company approach. In the above papers, only the initial capital tends to infinity. We also consider a fixed time period which should be well motivated from the practical point of view. This line is also taken in De Kok [11] and Tang and Tsitsiashvili [21]. Finally, our results seem to allow general models for the underlying processes. A typical assumption in related papers is that the pairs $(Y_{n1}, \xi_1), \ldots, (Y_{nd}, \xi_d)$ are independent and identically distributed.

Example 4.1. Let $d = 2$ and

$$\Gamma(\lambda_1, \lambda_2) = \mu_1\lambda_1 + \mu_2\lambda_2 + \frac{\sigma_1^2\lambda_1^2}{2} + \frac{\sigma_2^2\lambda_2^2}{2} \tag{4.5}$$

for $\lambda_1, \lambda_2 \in \mathbf{R}$ where $\mu_1, \mu_2 \in (-\infty, 0)$ and $\sigma_1, \sigma_2 \in (0, \infty)$ are constants. For example, $(Y_{n1}, Y_{n2})^{\mathrm{T}}$ could have an appropriate normal distribution for $n \in \mathbf{N}$. This assumption is not very usual in this context but is made here in order to obtain the results in closed form. Clearly, Γ is continuous everywhere so that we can work with the function I_2 of (2.5). We have

$$\Gamma^*(x_1, x_2) = \frac{(x_1 - \mu_1)^2}{2\sigma_1^2} + \frac{(x_2 - \mu_2)^2}{2\sigma_2^2} \tag{4.6}$$

for $x_1, x_2 \in \mathbf{R}$. Let $\mathcal{S}_j$ be the support of the distribution of ξ_j for $j = 1, 2$. We assume that $\mathcal{S}_j = [\alpha_j, \beta_j]$ where $0 < \alpha_j \leq \beta_j < \infty$. Assume further that the support of (ξ_1, ξ_2) is $\mathcal{S}_1 \times \mathcal{S}_2$. Let

$$A = \begin{pmatrix} a_1 & 0 \\ a_1 & a_1a_2 \end{pmatrix} \in \mathcal{S}. \tag{4.7}$$

Obviously, A is invertible so that

$$\left(\Gamma A^{\mathrm{T}}\right)^*(y) = \Gamma^*\left(A^{-1}y\right) = \frac{1}{2\sigma_1^2}\left(\frac{y_1}{a_1} - \mu_1\right)^2 + \frac{1}{2\sigma_2^2}\left(\frac{y_2 - y_1}{a_1a_2} - \mu_2\right)^2 \tag{4.8}$$

for every $y = (y_1, y_2)^{\mathrm{T}} \in \mathbf{R}^2$.

We will derive estimates for the ruin probability $\mathbf{P}(T \leq 2)$. To get lower bounds, we have to minimize I_2 over the set C_u of (4.4), and to get upper bounds, over the set $\mathrm{cl}C_u$. It is clear that the infima will be equal. We will consider $\mathrm{cl}C_u$ in the sequel. Thus we have to minimize (4.8) over the set

$$\{a_1 \in [\alpha_1, \beta_1], a_2 \in [\alpha_2, \beta_2], \{y_1 \geq u \text{ or } y_2 \geq u\}\}. \tag{4.9}$$

For $y_1 \geq u$, we can always choose y_2 such that the second term on the right hand side of (4.8) vanishes. Hence,

$$\inf\left\{\left(\Gamma A^{\mathrm{T}}\right)^*(y); a_1 \in [\alpha_1, \beta_1], a_2 \in [\alpha_2, \beta_2], y_1 \geq u\right\} = \frac{1}{2\sigma_1^2}\left(\frac{u}{\beta_1} - \mu_1\right)^2. \tag{4.10}$$

Let now $y_2 \geq u$. The optimum for $y_1 \geq u$ was found in (4.10) so that we can assume that $y_1 < u$. Obviously, we have to take $y_2 = u$ and $a_2 = \beta_2$. By a straightforward calculation, it is seen that for every given $a_1 \in \mathcal{S}_1$, the right choice for y_1 is

$$y_1 = \frac{\sigma_1^2 u}{\sigma_1^2 + \sigma_2^2\beta_2^2} + \frac{\sigma_2^2 a_1\beta_2^2\mu_1 - \sigma_1^2 a_1\beta_2\mu_2}{\sigma_1^2 + \sigma_2^2\beta_2^2}. \tag{4.11}$$

By substituting this, we end up by minimizing

$$\frac{1}{2(\sigma_1^2+\sigma_2^2\beta_2^2)}\left(\frac{u}{a_1}-\mu_1-\beta_2\mu_2\right)^2 \tag{4.12}$$

over a_1. Hence, $a_1=\beta_1$ gives the optimum. By combining the results, we get

$$\lim_{n\to\infty} n^{-1}\log \mathbf{P}(T\le 2) = -\min\left\{\frac{1}{2\sigma_1^2}\left(\frac{u}{\beta_1}-\mu_1\right)^2, \frac{1}{2(\sigma_1^2+\sigma_2^2\beta_2^2)}\left(\frac{u}{\beta_1}-\mu_1-\beta_2\mu_2\right)^2\right\}. \tag{4.13}$$

It is not surprising that a_1 and a_2 are maximal in the solution of the above optimization problem. Namely, large values of the discount factors correspond to bad returns on the investments.

Limits like (4.13) provide a tool for the risk management, at least in a crude sense. The company may fix the target value for (4.13) which can be seen as a solvency requirement. Then the parameters can be chosen in an optimal way, for example, such that the expected profit will be maximal at the end of the year 2. This could be done by making use of appropriate options to affect the supports $[\alpha_j,\beta_j]$, $j=1,2$. Intuitively, it seems that the terms under the minimum in (4.13) should be equal at the optimum. Similar balancing can be found in Martin-Löf [12] in a different but related context.

5. Proofs

Proof of Theorem 2.1. Consider (2.6). Let $F'\subseteq \mathbf{R}^{d'}$ be closed and

$$H=\bigcup_{A\in\mathcal{S}}\{x\in\mathbf{R}^d;\ Ax\in F'\}. \tag{5.1}$$

We begin by showing that H is closed. Let $x\in H^c$ be fixed. Then for a given $A\in\mathcal{S}$, we have $Ax\in(F')^c$. Further, there exists $\varepsilon_A>0$ such that $A'x'\in(F')^c$ whenever A' and x' are such that $|A'-A|<\varepsilon_A$ and $|x'-x|<\varepsilon_A$ (the metrics are Euclidean). The open balls $B(A,\varepsilon_A)$, $A\in\mathcal{S}$, cover the compact set $\mathcal{S}$. Thus we can extract a finite number of balls, $B(A_1,\varepsilon_{A_1}),\ldots,B(A_N,\varepsilon_{A_N})$, say, such that

$$\mathcal{S}\subseteq\bigcup_{k=1}^{N}B(A_k,\varepsilon_{A_k}). \tag{5.2}$$

Let $\varepsilon=\min\{\varepsilon_{A_1},\ldots,\varepsilon_{A_N}\}$. If $|x'-x|<\varepsilon$ then $A'x'\in(F')^c$ for every $A'\in\mathcal{S}$. Thus $B(x,\varepsilon)\subseteq H^c$ so that H is closed.

Obviously,

$$\mathbf{P}(\mathcal{A}X_n\in F')=\mathbf{P}(\mathcal{A}X_n\in F',\mathcal{A}\in\mathcal{S})\le\mathbf{P}(X_n\in H). \tag{5.3}$$

By the large deviations upper bounds,

$$\limsup_{n\to\infty} n^{-1}\log\mathbf{P}(\mathcal{A}X_n\in F')\le-\inf\{\Gamma^*(x);\ x\in H\}. \tag{5.4}$$

Let $x\in H$ be arbitrary. We next show that there exist $A\in\mathcal{S}$ and $y\in F'$ such that

$$\Gamma^*(x)\ge((\mathrm{cl}\Gamma)A^{\mathrm{T}})^*(y). \tag{5.5}$$

Choose $A \in \mathcal{S}$ such that $Ax \in F'$ and take $y = Ax$. Recall the definition of $A\Gamma^*$ from the Appendix. By (A.4),

$$((\mathrm{cl}\Gamma)A^{\mathrm{T}})^*(y) = \mathrm{cl}(A\Gamma^*)(Ax) \\ \le (A\Gamma^*)(Ax) = \inf\{\Gamma^*(v); v \in \mathbf{R}^d, Av = Ax\} \le \Gamma^*(x). \tag{5.6}$$

Thus we have (5.5). It follows that

$$\inf\{\Gamma^*(x); x \in H\} \ge \inf_{y \in F'} \inf_{A \in \mathcal{S}} ((\mathrm{cl}\Gamma)A^{\mathrm{T}})^*(y) = \inf_{y \in F'} I_1(y). \tag{5.7}$$

This and (5.4) imply (2.6).

Consider (2.7). We have $\mathrm{cl}\Gamma \le \Gamma$ so that $I_2 \le I_1$ Thus (2.7) follows from (2.6). This completes the proof of Theorem 2.1. □

We state a technical lemma before the proof of Theorem 2.2.

Lemma 5.1. *Assume the conditions of Theorem 2.2. Let $\alpha \in \mathbf{R}$ and*

$$\Psi(\alpha) = \{x \in \mathbf{R}^d; \Gamma^*(x) \le \alpha\}. \tag{5.8}$$

Then

$$\limsup_{n\to\infty} n^{-1} \log \mathbf{P}(X_n \in \Psi(\alpha)^c) \le -\alpha. \tag{5.9}$$

Proof of Lemma 5.1. Define

$$K = \{x \in \mathbf{R}^d; \Gamma^*(x) \in (\alpha, \infty)\}. \tag{5.10}$$

We have $\Gamma^*(x) \in [\alpha, \infty]$ for every $x \in \mathrm{cl}K$ because Γ^* is continuous on $\mathrm{dom}\,\Gamma^*$. By the large deviations upper bounds,

$$\limsup_{n\to\infty} n^{-1} \log \mathbf{P}(X_n \in \mathrm{cl}K) \le -\alpha. \tag{5.11}$$

Consider an $x \in (\mathrm{dom}\,\Gamma^*)^c \cap \mathrm{cl}(\mathrm{dom}\,\Gamma^*)$. Let $\{x_n\} \subseteq \mathrm{dom}\,\Gamma^*$ be a sequence which converges to x. It follows from the lower semicontinuity of Γ^* that

$$\liminf_{n\to\infty} \Gamma^*(x_n) \ge \Gamma^*(x) = \infty. \tag{5.12}$$

It is seen that $x_n \in K$ for large n so that $x \in \mathrm{cl}K$. Hence,

$$\Psi(\alpha)^c \cap \mathrm{cl}(\mathrm{dom}\,\Gamma^*) = (K \cup (\mathrm{dom}\,\Gamma^*)^c) \cap \mathrm{cl}(\mathrm{dom}\,\Gamma^*) \\ = (K \cap \mathrm{cl}(\mathrm{dom}\,\Gamma^*)) \cup ((\mathrm{dom}\,\Gamma^*)^c \cap \mathrm{cl}(\mathrm{dom}\,\Gamma^*)) \subseteq \mathrm{cl}K. \tag{5.13}$$

Thus

$$\Psi(\alpha)^c = (\Psi(\alpha)^c \cap \mathrm{cl}(\mathrm{dom}\,\Gamma^*)) \cup (\Psi(\alpha)^c \cap (\mathrm{cl}(\mathrm{dom}\,\Gamma^*))^c) \\ \subseteq (\mathrm{cl}K) \cup (\mathrm{cl}(\mathrm{dom}\,\Gamma^*))^c. \tag{5.14}$$

We obtain (5.9) by (5.11) and (2.8) and Lemma 1.2.15 of Dembo and Zeitouni [4]. □

Proof of Theorem 2.2. As observed at the end of the proof of Theorem 2.1, it suffices to show that (2.6) holds. Let $F' \subseteq \mathbf{R}^{d'}$ be closed and let H be as in (5.1). For $\alpha \in \mathbf{R}$, let $\Psi(\alpha)$ be as in Lemma 5.1. Then by (5.3),

$$\mathbf{P}(\mathcal{A}X_n \in F') \le \mathbf{P}(\{X_n \in \mathrm{cl}(\Psi(\alpha) \cap H)\} \cup \{X_n \in \Psi(\alpha)^c\}). \tag{5.15}$$

By the large deviations upper bounds,

$$\limsup_{n\to\infty} n^{-1} \log \mathbf{P}(X_n \in \mathrm{cl}(\Psi(\alpha) \cap H)) \le -\inf\{\Gamma^*(x); x \in \mathrm{cl}(\Psi(\alpha) \cap H)\}. \tag{5.16}$$

Let $x \in \mathrm{cl}(\Psi(\alpha) \cap H)$ and let $\{x_n\} \subseteq \Psi(\alpha) \cap H$ be a sequence which converges to x. For every $n \in \mathbf{N}$, we conclude as in (5.6) that

$$\Gamma^*(x_n) \ge ((\mathrm{cl}\Gamma)A_n^{\mathrm{T}})^*(y_n) \tag{5.17}$$

for some $A_n \in \mathcal{S}$ and $y_n \in F'$. By our assumptions, either $\Gamma^*(x) = \infty$ or, otherwise, $\Gamma^*(x_n)$ tends to $\Gamma^*(x)$ when n tends to infinity. It follows that

$$\Gamma^*(x) \ge \inf_{n\in\mathbf{N}} \Gamma^*(x_n) \ge \inf_{y\in F'} \inf_{A\in\mathcal{S}} ((\mathrm{cl}\Gamma)A^{\mathrm{T}})^*(y). \tag{5.18}$$

Consequently,

$$\inf\{\Gamma^*(x); x \in \mathrm{cl}(\Psi(\alpha) \cap H)\} \ge \inf_{y\in F'} \inf_{A\in\mathcal{S}} ((\mathrm{cl}\Gamma)A^{\mathrm{T}})^*(y) = \inf_{y\in F'} I_1(y).$$

By (5.15) and (5.16) and Lemma 5.1 and by Lemma 1.2.15 of Dembo and Zeitouni [4],

$$\limsup_{n\to\infty} n^{-1} \log \mathbf{P}(\mathcal{A}X_n \in F') \le -\min\left(\alpha, \inf_{y\in F'} I_1(y)\right). \tag{5.19}$$

We obtain (2.6) by letting α tend to infinity. □

Proof of Theorem 2.3. Consider (2.9). Let $A, A_1, A_2, \ldots$ be arbitrary deterministic $d' \times d$ matrices such that $A_n \to A$ in the Euclidean metric when $n \to \infty$. By Theorem 2.1 of Dinwoodie and Zabell [5], it suffices to show that for every open set $G' \subseteq \mathbf{R}^{d'}$,

$$\liminf_{n\to\infty} n^{-1} \log \mathbf{P}(A_n X_n \in G') \ge -\inf_{y\in G'} ((\mathrm{cl}\Gamma)A^{\mathrm{T}})^*(y). \tag{5.20}$$

Let $\varepsilon, \varepsilon' > 0$ and let $y \in \mathbf{R}^{d'}$ be arbitrary such that $((\mathrm{cl}\Gamma)A^{\mathrm{T}})^*(y) < \infty$. For (5.20), it is sufficient to show that for the open ball $B(y, \varepsilon)$, we have

$$\liminf_{n\to\infty} n^{-1} \log \mathbf{P}(A_n X_n \in B(y, \varepsilon)) \ge -((\mathrm{cl}\Gamma)A^{\mathrm{T}})^*(y). \tag{5.21}$$

By (A.4), $((\mathrm{cl}\Gamma)A^{\mathrm{T}})^* = \mathrm{cl}(A\Gamma^*)$. By Theorem 7.5 of Rockafellar [20], there exists $y_{\varepsilon'} \in \mathbf{R}^{d'}$ such that $|y - y_{\varepsilon'}| \le \varepsilon/4$ and

$$(A\Gamma^*)(y_{\varepsilon'}) \le \mathrm{cl}(A\Gamma^*)(y) + \varepsilon' = ((\mathrm{cl}\Gamma)A^{\mathrm{T}})^*(y) + \varepsilon'. \tag{5.22}$$

Thus there exists $x_{\varepsilon'} \in \mathbf{R}^d$ such that $Ax_{\varepsilon'} = y_{\varepsilon'}$ and

$$\Gamma^*(x_{\varepsilon'}) \le ((\mathrm{cl}\Gamma)A^{\mathrm{T}})^*(y) + 2\varepsilon'. \tag{5.23}$$

Fix $\delta > 0$ such that

$$B(x_{\varepsilon'}, \delta) \subseteq \{x \in \mathbf{R}^d; Ax \in B(y, \varepsilon/2)\}. \tag{5.24}$$

Then for large n,

$$\mathbf{P}(A_n X_n \in B(y, \varepsilon)) \ge \mathbf{P}(X_n \in B(x_{\varepsilon'}, \delta)). \tag{5.25}$$

By the large deviations lower bounds and by (5.23),

$$\liminf_{n\to\infty} n^{-1} \log \mathbf{P}(A_n X_n \in B(y, \varepsilon)) \geq -\Gamma^*(x_{\varepsilon'}) \\ \geq -((\mathrm{cl}\Gamma)A^{\mathrm{T}})^*(y) - 2\varepsilon'. \tag{5.26}$$

This implies (5.21).

Consider (2.10). Our assumptions together with (A.5) imply that $((\mathrm{cl}\Gamma)A^{\mathrm{T}})^* = (\Gamma A^{\mathrm{T}})^*$ for every matrix $A \in \mathcal{S}$. Thus (2.10) follows from (2.9). □

Proof of Theorem 3.1. The upper bounds of the theorem are well known. Consider the lower bounds. We recall the main steps from the proof of Theorem 2.3.6 in Dembo and Zeitouni [4] and give necessary complementary observations. Let $\lambda_0 \in \mathrm{int}(\mathrm{dom}\,\Lambda)$. Then

$$\liminf_{n\to\infty} n^{-1} \log \mathbf{P}(X_n \in B(\nabla\Lambda(\lambda_0), \varepsilon)) \geq -\Lambda^*(\nabla\Lambda(\lambda_0)) \tag{5.27}$$

for every $\varepsilon > 0$. We next show that

$$\mathrm{ri}(\mathrm{dom}\,\Lambda^*) \subseteq \nabla\Lambda(\mathrm{int}(\mathrm{dom}\,\Lambda)). \tag{5.28}$$

This is a standard step in the proof of the Gärtner–Ellis theorem but needs a justification here because we do not assume that Λ is lower semicontinuous. By Rockafellar [20, Theorem 7.4], we have $(\mathrm{cl}\Lambda)(\lambda) = \Lambda(\lambda)$ except perhaps for boundary points λ of $\mathrm{dom}\,\Lambda$. Thus $\mathrm{cl}\Lambda$ has the same gradients as Λ and $\mathrm{cl}\Lambda$ is also essentially smooth. Further, the interiors of the effective domains of $\mathrm{cl}\Lambda$ and Λ are equal. We have (5.28) when Λ is replaced by $\mathrm{cl}\Lambda$. See Rockafellar [20, Corollary 26.4.1], or Dembo and Zeitouni [4, Lemma 2.3.12]. By (A.6) and the above discussion, nothing changes in (5.28) when Λ is replaced by $\mathrm{cl}\Lambda$. Thus it holds true. The lower bounds in question now follow from (5.27) and from the fact that for every open set $G \subseteq \mathbf{R}^d$,

$$\inf\{\Lambda^*(x); x \in G\} = \inf\{\Lambda^*(x); x \in G \cap \mathrm{ri}(\mathrm{dom}\,\Lambda^*)\}. \quad \square \tag{5.29}$$

Appendix

We recall here some concepts and results from the theory of convex functions. Let $B \subseteq \mathbf{R}^d$ be convex. The interior of B is denoted by $\mathrm{int}B$. The *relative interior* of B is the interior of B which results when B is considered as a subset of its affine hull. It is denoted by $\mathrm{ri}B$. Let $f : \mathbf{R}^d \to \mathbf{R} \cup \{\pm\infty\}$ be a convex function. The *effective domain* $\mathrm{dom}\, f$ of f is by definition,

$$\mathrm{dom}\, f = \{\lambda \in \mathbf{R}^d; f(\lambda) < \infty\}. \tag{A.1}$$

We call f *essentially smooth* if $\mathrm{int}(\mathrm{dom}\, f)$ is non-empty, f is differentiable on $\mathrm{int}(\mathrm{dom}\, f)$ and $|\nabla f(\lambda_n)|$ tends to infinity for every sequence $\{\lambda_n\} \subseteq \mathrm{int}(\mathrm{dom}\, f)$ tending to a boundary point of $\mathrm{dom}\, f$. Denote by $\mathrm{cl}f$ the *closure* of f. That is, $\mathrm{cl}f$ is the greatest lower semicontinuous function majorized by f if $f(\lambda) > -\infty$ for every $\lambda \in \mathbf{R}^d$, and otherwise, $\mathrm{cl}f$ equals $-\infty$ everywhere. In the former case, $\mathrm{cl}f$ is actually the *lower semicontinuous hull* of f. Let $g : \mathbf{R}^{d'} \to \mathbf{R} \cup \{\pm\infty\}$ be a convex function and let A be a $d' \times d$ matrix. Recall the definition of gA from Section 2. Define the function $Af : \mathbf{R}^{d'} \to \mathbf{R} \cup \{\pm\infty\}$ by

$$(Af)(\kappa) = \inf\{f(\lambda); \lambda \in \mathbf{R}^d, A\lambda = \kappa\}. \tag{A.2}$$

Then Af is convex. For the Fenchel–Legendre transforms, we have

$$(Af)^* = f^* A^{\mathrm{T}} \tag{A.3}$$

and

$$((\mathrm{cl}\, g)A)^* = \mathrm{cl}(A^{\mathrm{T}} g^*). \tag{A.4}$$

If $A\lambda \in \mathrm{ri}(\mathrm{dom}\, g)$ for some $\lambda \in \mathbf{R}^d$ then

$$((\mathrm{cl}\, g)A)^* = (gA)^*. \tag{A.5}$$

Finally,

$$(\mathrm{cl}\, f)^* = f^* \quad \text{and} \quad (f^*)^* = \mathrm{cl}\, f. \tag{A.6}$$

The proofs can be found in Rockafellar [20]. The convexity of Af is stated in Theorem 5.7. Results (A.3)–(A.5) are included in Theorem 16.3. The background for (A.6) is presented in Theorem 12.2.

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